Chapter 5
Pile Groups

1. Design Considerations

This chapter provides several hand calculation methods for a quick estimate of the capacity and movement characteristics of a selected group of driven piles or drilled shafts for given soil conditions. A computer assisted method such as described in Chapter 5, paragraph 4, is recommended for a detailed solution of the performance of driven pile groups. Recommended factors of safety for pile groups are also given in Table 3-2. Calculation of the distribution of loads in a pile group is considered in paragraph 2b, Chapter 2.

a. Driven piles. Driven piles are normally placed in groups with spacings less than 6*B* where *B* is the width or diameter of an individual pile. The pile group is often joined at the ground surface by a concrete slab such as a pile cap, Figure 5-1a. If pile spacing within the optimum range, the load capacity of groups of driven piles in cohesionless soils can often be greater than the sum of the capacitites of isolated piles, because driving can compact sands and can increase skin friction and end-bearing resistance.

b. Drilled shafts. Drilled shafts are often not placed in closely spaced groups, Figure 5-1b, because these foundations can be constructed with large diameters and can extend to great depths. Exceptions include using drilled shafts as retaining walls or to improve the soil by replacing existing soil with multiple drilled shafts. Boreholes prepared for construction of drilled shafts reduce effective stresses in soil adjacent to the sides and bases of shafts already in place. The load capacity of drilled shafts in cohesionless soils spaced less than 6*B* may therefore be less than the sum of the capacities of the individual shafts. For end-bearing drilled shafts, spacing of less than 6*B* can be used without significant reduction in load capacity.

2. Factors Influencing Pile Group Behavior

Piles are normally constructed in groups of vertical, batter, or a combination of vertical and batter piles. The distribution of loads applied to a pile group are transferred nonlinearly and indeterminately to the soil. Interaction effects between adjacent piles in a group lead to complex solutions. Factors considered below affect the resistance of the pile group to movement and load transfer through the pile group to the soil.

a. Soil modulus. The elastic soil modulus E_s and the lateral modulus of subgrade reaction E_{ls} relate lateral, axial, and rotational resistance of the pile-soil medium to displacements. Water table depth and seepage pressures affect the modulus of cohesionless soil. The modulus of submerged sands should be reduced by the ratio of the submerged unit weight divided by the soil unit weight.

b. Batter. Battered piles are used in groups of at least two or more piles to increase capacity and loading resistance. The angle of inclination should rarely exceed 20 degrees from the vertical for normal construction and should never exceed 26½ degrees. Battered piles should be avoided where significant negative skin friction and downdrag forces may occur. Batter piles should be avoided where the structure's foundation must respond with ductility to unusually large loads or where large seismic loads can be transferred to the structure through the foundation.

c. Fixity. The fixity of the pile head into the pile cap influences the loading capacity of the pile group. Fixing the pile rather than pinning into the pile cap usually increases the lateral stiffness of the group, and the moment. A group of fixed piles can therefore support about twice the lateral load at identical deflections as the pinned group. A fixed connection between the pile and cap is also able to transfer significant bending moment through the connection. The minimum vertical embedment distance of the top of the pile into the cap required for achieving a fixed connection is 2*B* where *B* is the pile diameter or width.

d. Stiffness of pile cap. The stiffness of the pile cap will influence the distribution of structural loads to the individual piles. The thickness of the pile cap must be at least four times the width of an individual pile to cause a significant influence on the stiffness of the foundation (Fleming et al. 1985). A ridgid cap can be assumed if the stiffness of the cap is 10 or more times greater than the stiffness of the individual piles, as generally true for massive concrete caps. A rigid cap can usually be assumed for gravity type hydraulic structures.

e. Nature of loading. Static, cyclic, dynamic, and transient loads affect the ability of the pile group to resist the applied forces. Cyclic, vibratory, or repeated static loads cause greater displacements than a sustained static load of the same magitude. Displacements can double in some cases.

f. Driving. The apparent stiffness of a pile in a group may be greater than that of an isolated pile driven in cohesionless soil because the density of the soil within and around a pile group can be increased by driving. The pile group as a whole may not reflect this increased stiffness because the soil around and outside the group may not be favorably affected by driving and displacements larger than anticipated may occur.

g. Sheet pile cutoffs. Sheet pile cutoffs enclosing a pile group may change the stress distribution in the soil and influence the group load capacity. The length of the cutoff should be determined from a flow net or other seepage analysis. The net pressure acting on the cutoff is the sum of the unbalanced earth and water pressures caused by the

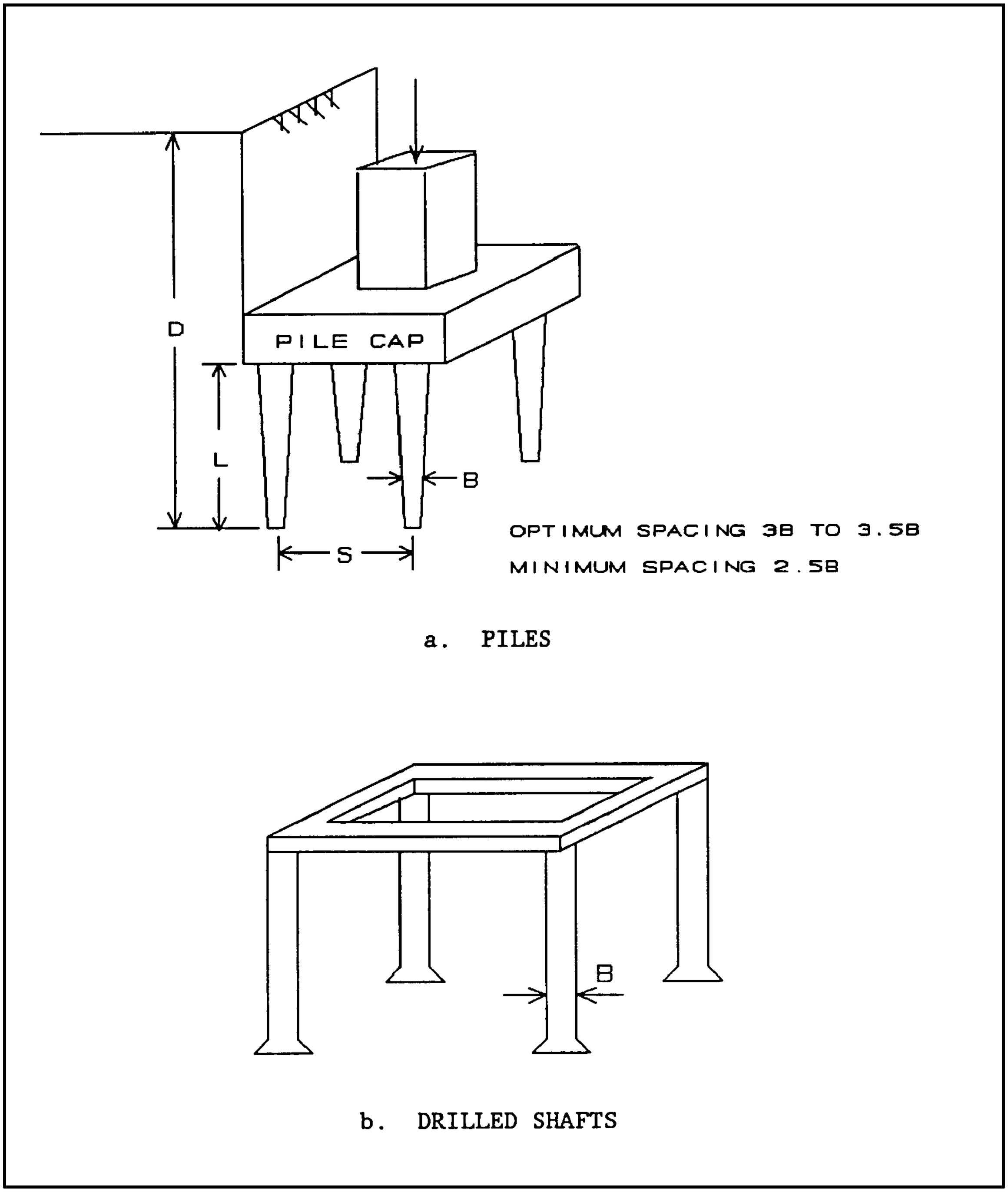


Figure 5-1. Groups of deep foundations

cutoff. Steel pile cutoffs should be considered in the analysis as not totally impervious. Flexible steel sheet piles should cause negligible load to be transferred to the soil. Rigid cutoffs, such as a concrete cutoff, will transfer the unbalanced earth and water pressures to the structure and shall be accounted for in the analysis of the pile group.

h. Interaction effects. Deep foundations where spacings between individual piles are less than six times the pile width *B* cause interaction effects between adjacent piles from overlapping of stress zones in the soil, Figure 5-2. In situ soil stresses from pile loads are applied over a much larger area and extend to a greater depth leading to greater settlement.

i. Pile spacing. Piles in a group should be spaced so that the bearing capacity of the group is optimum. The optimum spacing for driven piles is 3 to 3.5B (Vesic 1977) or 0.02L + 2.5B, where L is the embedded length of the piles (Canadian Geotechnical Society 1985). Pile spacings should be at least 2.5B.

3. Design for Vertical Loads

The methodology should provide calculations of the pile group capacity and displacements such that the forces are in equilibrium between the structure and the supporting piles and between the piles and soil supporting the piles. The allowable group capacity is the ultimate group capacity divided by the factor of safety. The factor of safety is usually 3 for pile groups, Table 3-2. Methods for analysis of axial load capacity and settlement are provided below.

a. Axial capacity of drilled shaft groups. The calculation depends on whether the group is in sands or clays. Installation in cohesionless sands causes stress relief and a reduced density of the sands during construction. The efficiency method is appropriate whether the pile cap is or is not in firm contact with the ground. Block failure, however, may occur when the base of the group overlies soil that is much weaker than the soil at the base of the piles. Group capacity in cohesive soil depends on whether or not the pile cap is in contact with the ground.

(1) Group capacity for cohesionless soil. Group ultimate capacity is calculated by the efficiency method for cohesionless soil

$$Q_{ug} = n \times E_g \times Q_u \tag{5-1}$$

where

Q_{ug} = group capacity, kips

n = number of shafts in the group

E_g = efficiency

Q_u = ultimate capacity of the single shaft

E_g should be > 0.7 for spacings = 3B and increases linearly to 1.0 for spacings = 6B where B is the shaft diameter or width (FHWA-HI-88-042). E_g should vary linearly for spacings between 3B and 6B. E_g = 0.7 for spacings ≤ 3B. The factor of safety of the group is the same as that of the individual shafts.

(2) Group capacity for cohesive soil. Groups with the cap in firm contact with the clay may fail as a block of soil containing the drilled shafts, even at large spacings between individual shafts. The ultimate group capacity is either the lesser of the sum of the individual capacities or the ultimate capacity of the block. The block capacity is determined by

$$Q_{ug} = 2L\,(H_L + H_W)\,C_{ua} + N_{cg} \times C_{ub} \times H_L \times H_W \tag{5-2}$$

where

L = depth of penetration meter (feet)

H_L = horizontal length of group meter (feet)

H_W = horizontal width of group meter (feet)

C_{ua} = average undrained shear strength of cohesive soil in which the group is placed kN/m² (ksf)

C_{ub} = undrained shear strength of cohesive soil at the base kN/m² (ksf)

N_{cg} = cohesion group bearing capacity factor

N_{cg} is determined by

$$N_{cg} = 5\left(1 + 0.2\frac{H_W}{H_L}\right)\left(1 + 0.2\frac{L}{H_W}\right) \quad \text{for } \frac{L}{H_W} \le 2.5 \tag{5-3a}$$

$$N_{cg} = 7.5\left(1 + 0.2\frac{H_W}{H_L}\right) \quad \text{for } \frac{L}{H_W} > 2.5 \tag{5-3b}$$

The group capacity is calculated by the efficiency equation 5-1 if the pile cap is not in firm contact with the soil. Overconsolidated and insensitive clay shall be treated as if the cap is in firm contact with the ground.

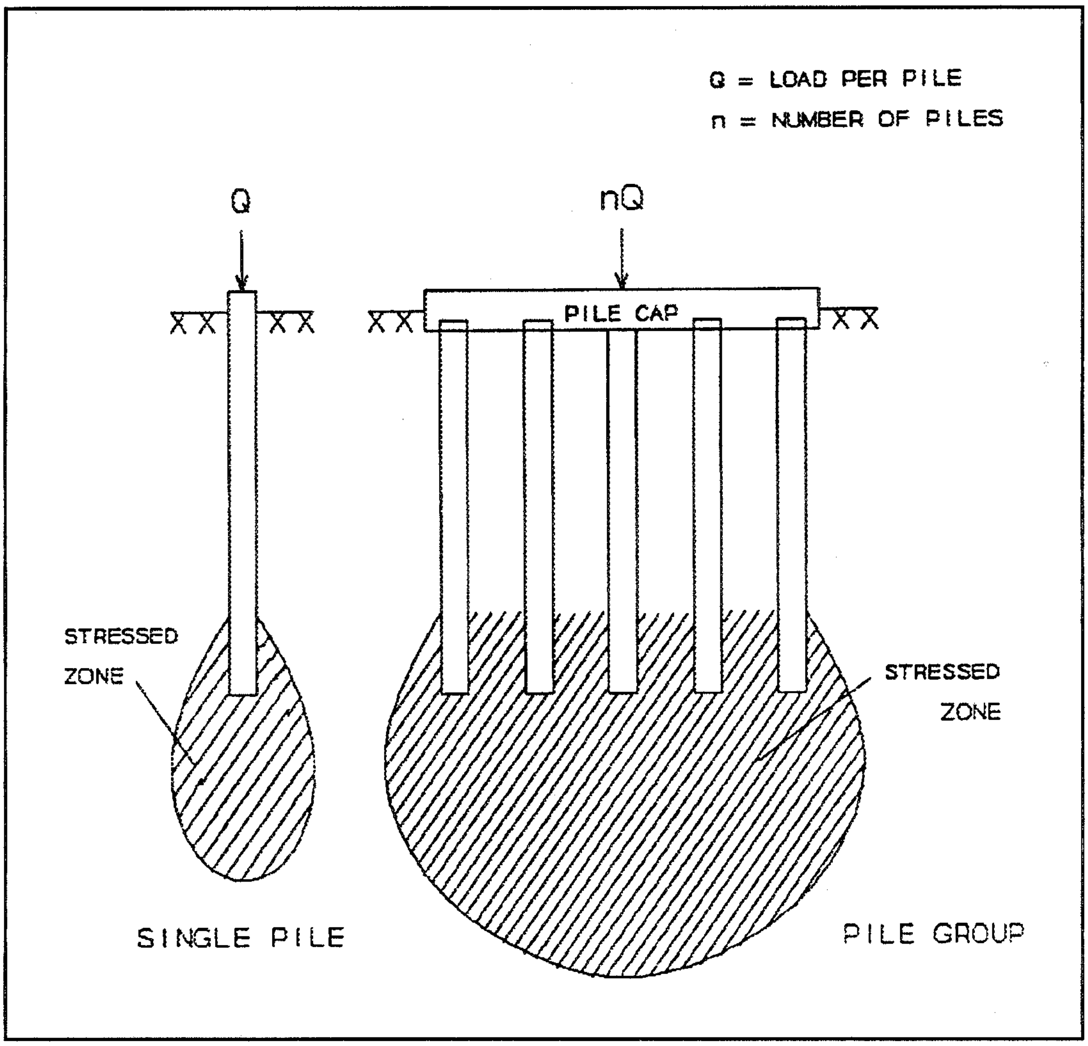


Figure 5-2. Stress zones in soil supporting piles

(a) Presence of locally soft soil should be checked because this soil may cause some driven piles or drilled shafts to fail. The equivalent mat method in Table 5-1 is recommended to calculate group capacity in soft clays, e.g. $C_u \leq 0.5$ ksf.

(b) The ultimate capacity of a group in a strong clay soil overlying weak clay may be estimated by assuming block punching through the weak underlying soil layer. Group capacity may be calculated by equation 5-2 using the undrained strength C_{ub} of the underlying weak clay. A less conservative solution is provided (FHWA-HI-88-042) by

$$Q_{ug} = Q_{ug,1} + \frac{Z_b}{10H_W}[Q_{ug,u} - Q_{ug,1}] \leq Q_{ug,u} \qquad (5\text{-}4)$$

where

$Q_{ug,l}$ = group capacity if base at top of lower weak soil, kips

$Q_{ug,u}$ = group capacity in the upper soil if the weaker lower soil were not present, kips

Z_b = vertical distance from the base of the shafts in the group to the top of the weak layer, feet

H_W = least width of group, feet

Equation 5-4 can also be used to estimate the ultimate capacity of a group in a strong cohesionless soil overlying a weak cohesive layer.

b. Axial capacity of driven pile groups. Driven piles are normally placed in groups with spacings less than $3B$ and joined at the ground surface by a concrete cap.

(1) Group capacity for cohesionless soil. Pile driving compacts the soil and increases end-bearing and skin friction resistance. Therefore, the ultimate group capacity of driven piles with spacings less than $3B$ can be greater than the sum of the capacities of the individual piles.

(2) Group capacity for cohesive soil. For this case, the ultimate capacity of a pile group is the lesser of the sum of the capacities of the individual piles or the capacity by block failure.

(a) The capacity of block failure is given by equation 5-2.

(b) The capacity of a pile group with the pile cap not in firm contact with the ground may be calculated by the efficiency method in equation 5-1.

(3) Uplift capacity. The ultimate uplift capacity of a pile group is taken as the lesser of the sum of the individual pile uplift capacities or the uplift capacity of the group considered as a block.

(a) Cohesionless soil. The side friction of pile groups in sands decreases with time if the piles are subject to vibration or lateral loads. The uplift capacity will be at least the weight of the soil and piles of the group considered as a block.

(b) Cohesive soil. The uplift capacity will include side friction and is estimated by

$$Q_{ug} = 2L\,(H_W + H_L)\,C_{ua} + W_g \qquad (5\text{-}5)$$

where

C_{ua} = average undrained shear strength along the perimeter of the piles, ksf

W_g = weight of the pile group considered as a block, kips

W_g also includes the weight of the soil within the group.

c. Settlement analysis. The settlement of a group of piles with load nQ (n - number of piles and Q = load per pile) can be much greater than the settlement of a single pile with load Q because the value of the stress zones of a pile group is much larger and extends deeper than that of a single pile, Figure 5-2. Hand calculation methods for estimating the settlement of pile groups are approximate. An estimate of settlement can also be obtained by considering the pile group as an equivalent mat as in Table 5-1, then calculating the settlement of this mat as given in chapter 5 of TM 5-818-1, "Soils and Geology; Procedures for Foundation Design of Buildings and Other Structures (Except Hydraulic Structures)."

(1) Immediate settlement. A simple method for estimating group settlement from the settlement of a single pile is to use a group settlement factor

$$\rho_g = g_f\,\rho \qquad (5\text{-}6a)$$

where

ρ_g = group settlement, feet

g_f = group settlement factor

ρ = settlement of single pile, feet

(a) The group settlement factor for sand (Pile Buck Inc. 1992) is

$$g_f = \left(\frac{H_W}{B}\right)^{0.5} \qquad (5\text{-}6b)$$

where H_W = width of the pile group and B is the pile diameter or width.

(b) The group settlement factor for clay (Pile Buck Inc. 1992) is

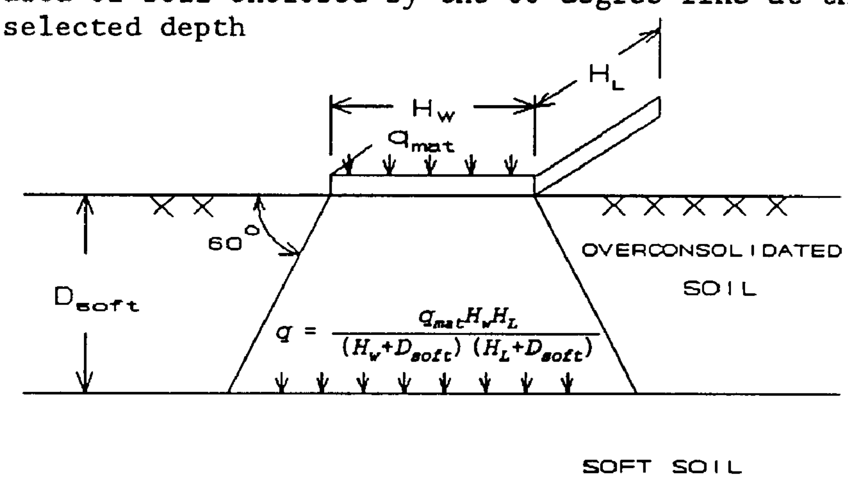
$$g_f = 1 + \sum_{i=1}^{n} \frac{B_i}{\pi s_i} \qquad (5\text{-}6c)$$

Table 5-1
Equivalent Mat Method of Group Pile Capacity Failure in Soft Clays

Step	Description
1	Replace group with a flexible mat of same dimensions as the group at some depth along the pile length; mat depth determined as follows:

Depth	Soil Condition
Ground Surface	Highly overconsolidated soil at the surface underlain by softer soil
2/3 of pile length from top	Group support obtained mostly from skin friction
Pile tip	End-bearing piles

Step	Description
2	Assume the mat carries the full group load
3	Distribute pressure on the mat to the underlying soft clay either by a line that makes a 60-degree angle with the horizontal or by Boussinesq theory; the 60-degree method reduces mat pressure by the ratio of mat area divided by area of soil enclosed by the 60-degree line at the selected depth



Step	Description
4	Compare the distributed pressure at the top of the soft clay with $9C_u$ where C_u is the average undrained shear strength of the soft clay

where n = number of piles in the group where

n = number of piles in the group

s_i = distance from pile i to the location in the group where group settlement is to be calculated, feet

(2) Estimates using field soil test results. Standard penetration and cone penetration test data can provide useful estimates assuming the group can be represented by an equivalent single pile.

(a) Settlement of pile groups in a homogeneous sand deposit not underlain by a more compressible soil at greater depth (Meyerhof 1976) is

$$\rho_g = \frac{q\sqrt{H_W}}{N_{spt}} I \qquad (5\text{-}7a)$$

$$I = 1 - \frac{L}{8H_W} \geq 0.5 \qquad (5\text{-}7b)$$

where

ρ_g = settlement of pile group, in.

q = net foundation pressure on the group, ksf

B_g = width of pile group, feet

I = influence factor

N_{spt} = average standard penetration resistance within the depth beneath the pile tip equal to the group width corrected to an effective overburden pressure of 2 kips per square foot, blows/feet

L = embedment depth of equivalent pile, feet

(b) The calculated settlement should be doubled for a silty sand.

(c) Maximum settlement estimated from static cone penetration tests (Meyerhof 1976) is

$$\rho_g = \frac{qH_W}{2q_c} I \qquad (5\text{-}7c)$$

where q_c is the average cone tip resistance within depth H_W beneath the pile tip in the same units as q.

(3) Consolidation settlement. Long-term settlement may be estimated for pile groups in clay by the equivalent method in Table 5-2.

Table 5-2
Equivalent Mat Method for Estimating Consolidation Settlement of Pile Groups in Clay

Step	Description
1	Replace the group with a mat at some depth along the embedded pile length L; this depth is 2/3 of L for friction piles and L for end bearing piles.
2	Distribute the load from the mat to the underlying soil by Boussinesq theory or the 60-degree method.
3	Calculate settlement of soil layers below the mat by one-dimensional consolidation theory; any soil above the mat is assumed incompressible.
4	Multiply the calculated settlement by 0.8 to account for rigidity of the group.

d. Application. A square three by three group of nine steel circular closed-end pipe piles with diameter B = 1.5 feet is to be driven to an embedment depth L = 30 feet in the same soils as Figure 3-15. These soils are a 15-feet layer of clay over sand. Spacing is 4B and the horizontial width H_W is 15B = 15 × 1.5 = 22.5 feet. The group upper- and lower-bound estimates of ultimate and allowable capacity and expected settlement at the allowable capacity are to be calculated to provide guidance for the pile group design. Pile driver analysis with a load test will be conducted at the start of construction. The factor of safety to be used for this analysis is 3.

(1) Group ultimate capacity. The group ultimate capacity Q_{ug} is expected to be the sum of the ultimate capacities of the individual piles. These piles are to be driven into sand which will densify and increase the end-bearing capacity. From Table 3-7, the calculated lower-bound ultimate capacity is $Q_{u,l}$ = 317 kips, and the upper-bound capacity is $Q_{u,u}$ = 520 kips. Therefore $Q_{ug,l}$ = $n \times Q_{u,l}$ = 9×317 = 2,853 kips and $Q_{ug,up}$ = 9×520 = 4,680 kips.

(2) Group allowable capacity. The allowable group upper- and lower-bound capacities are

$$Q_{ag,l} = \frac{Q_{ug,l}}{FS} = \frac{2,853}{3} = 951 \text{ kips}$$

$$Q_{ag,u} = \frac{Q_{ug,u}}{FS} = \frac{4,680}{3} = 1,560 \text{ kips}$$

The group allowable load is expected to be between 951 and 1,560 kips. Lower *FS* may be possible.

(3) Group settlement. Settlement at the allowable capacity will be greater than that of the individual piles. The settlement of each pile is to be initially determined from equation 3-38, then the group settlement is to be calculated from equation 5-6.

(a) The allowable lower- and upper-bound capacities of each individual pile is $Q_{a,l} = 317/3 = 106$ kips and $Q_{a,u} = 520/3 = 173$ kips. All the skin friction is assumed to be mobilized. Therefore, $Q_{s,l} = Q_{su,l} = 159$ kips $> Q_{a,l} = 106$ kips and $Q_{s,u} = Q_{su,u} = 231$ kips $> Q_{a,u} = 173$ kips. Base resistance will not be mobilized because the ultimate skin resistance Q_{su} exceeds the allowable capacity. From equation 3-38a, axial compression is

$$\rho_p = 12 \alpha_s Q_s \frac{L}{A E_p}$$

$$= 12 \times 0 \times 5 \times Q_s \frac{30}{\pi \times 1 \times 5^2 \times 432,000}$$

$$= 0.00006 \times Q_s \text{ inch}$$

The elastic modulus of the pile is assumed similar to concrete $E_p = 432,000$ ksf because this pile will be filled with concrete. Lower- and upper-bound axial compression is therefore

$$\rho_{p,l} = 0.00006 \times 106 = 0.0063 \text{ inch}$$

$$\rho_{p,u} = 0.00006 \times 173 = 0.0104 \text{ inch}$$

(b) Tip settlement from load transmitted along the shaft length from equation 3-38c is

$$\rho_s = \frac{12 C_s Q_s}{L q_{bu}} = \frac{12 \times 0.05 \times Q_s}{30 \times q_{bu}} = \frac{0.02 Q_s}{q_{bu}} \text{ inch}$$

where

C_s = [$0.93 + 0.16 (L/B_s)^{0.5}$] C_b = [$0.93 + 0.16 (30/1.5)^{0.5}$] (0.03) = 0.05

lower-bound $q_{bu,l} = 89$ ksf

upper-bound $q_{bu,u} = 163$ ksf from Table 3-7

Lower- and upper-bound tip settlement from the load transmitted along the shaft length for $Q_{s,l} = 138$ kips and $Q_{s,u} = 231$ kips is

$$\rho_{s,l} = \frac{0.02 \times 159}{103} = 0.031 \text{ inch}$$

$$\rho_{s,u} = \frac{0.02 \times 231}{163} = 0.028 \text{ inch}$$

Total settlements for lower- and upper-bound capacities are

$$\rho = \rho_b + \rho_s$$

$$\rho_{,l} = 0.006 + 0.036 = 0.042 \text{ inch}$$

$$\rho_{,u} = 0.010 + 0.028 = 0.038 \text{ inch}$$

Total settlement ρ is about 0.04 inch.

(c) Group settlement factor g_f from equation 5-6b is

$$g_f = \left(\frac{H_w}{B}\right)^{0.5} = \left(\frac{22.5}{1.5}\right)^{0.5} = 3.87$$

Group settlement from equation 5-6a is

$$\rho_g = g_f \rho = 3.87 \times 0.04 = 0.15 \text{ inch}$$

4. Design for Lateral Loads[1]

a. Response to lateral loading of pile groups. There are two general problems in the analysis of pile groups: the computation of the loads coming to each pile in the group and the determination of the efficiency of a group of closely spaced piles. Each of these problems will be discussed in the following paragraphs.

(1) Symmetric pile group. The methods that are presented are applicable to a pile group that is symmetrical about the line of action of the lateral load. That is, there is no twisting of the pile group so that no pile is subjected to torsion. Therefore, each pile in the group can undergo two translations and a rotation. However, the method that is presented for obtaining the distribution of loading to each pile can be extended to the general case where each pile can undergo three translations and three rotations (Reese, O'Neill, and Smith 1970; O'Neill, Ghazzaly, and Ha 1977; Bryant 1977).

(2) Soil reaction. In all of the analyses presented in this section, the assumption is made that the soil does not act against the pile cap. In many instances, of course, the pile cap is cast against the soil. However, it is possible that soil can settle away from the cap and that the piles will sustain the full load. Thus, it is conservative and perhaps logical to assume that the pile cap is ineffective in carrying any load.

(3) Pile spacing. If the piles that support a structure are spaced far enough apart that the stress transfer between them is minimal and if only shear loading is applied, the methods presented earlier in this manual can be employed. Kuthy et al. (1977) present an excellent treatment of this latter problem.

b. Widely spaced piles. The derivation of the equations presented in this section is based on the assumption that the piles are spaced far enough apart that there is no loss of efficiency; thus, the distribution of stress and deformation from a given pile to other piles in the group need not be considered. However, the method that is derived can be used with a group of closely spaced piles, but another level of interation will be required.

[1]Portions of this section were abstracted from the writings of Dr. L. C. Reese and his colleagues, with the permission of Dr. Reese.

(1) Model of the problem. The problem to be solved is shown in Figure 5-3. Three piles supporting a pile cap are shown. The piles may be of any size and placed on any batter and may have any penetration below the groundline. The bent may be supported by any number of piles but, as noted earlier, the piles are assumed to be placed far enough apart that each is 100 percent efficient. The soil and loading may have any characteristics for which the response of a single pile may be computed. The derivation of the necessary equations proceeds from consideration of a simplified structure such as that shown in Figure 5-4 (Reese and Matlock 1966; Reese 1966). The sign conventions for the loading and for the geometry are shown. A global coordinate system, a-b, is established with reference to the structure. A coordinate system, x-y, is established for each of the piles. For convenience in deriving the equilibrium equations for solution of the problem, the a-b axes are located so that all of the coordinates of the pile heads are positive. The soil is not shown, but as shown in Figure 5-4b, it is desirable to replace the piles with a set of "springs" (mechanisms) that represent the interaction between the piles and the supporting soil.

(2) Derivation of equations. If the global coordinate system translates horizontally Δh and vertically Δv and if the cordinate system, shown in Figure 5-4, rotates through the angle α_s, the movement of the head of each of the piles can be readily found. The angle α_s is assumed to be small in the derivation. The movement of a pile head x_t in the direction of the axis of the pile is

$$x_t = (\Delta h + b\alpha_s)\sin\theta + (\Delta v + a\alpha_s)\cos\theta \qquad (5\text{-}8)$$

The movement of a pile head y_t transverse to the direction of the axis of the pile (the lateral deflection) is

$$y_t = (\Delta h + b\alpha_s)\cos\theta - (\Delta v + a\alpha_s)\sin\theta \qquad (5\text{-}9)$$

The assumption is made in deriving equations 5-8 and 5-9 that the pile heads have the same relative positions in space before and after loading. However, if the pile heads move relative to each other, an adjustment can be made in equations 5-8 and 5-9 and a solution achieved by iteration. The movements computed by equations 5-8 and 5-9 will generate forces and moments at the pile head.

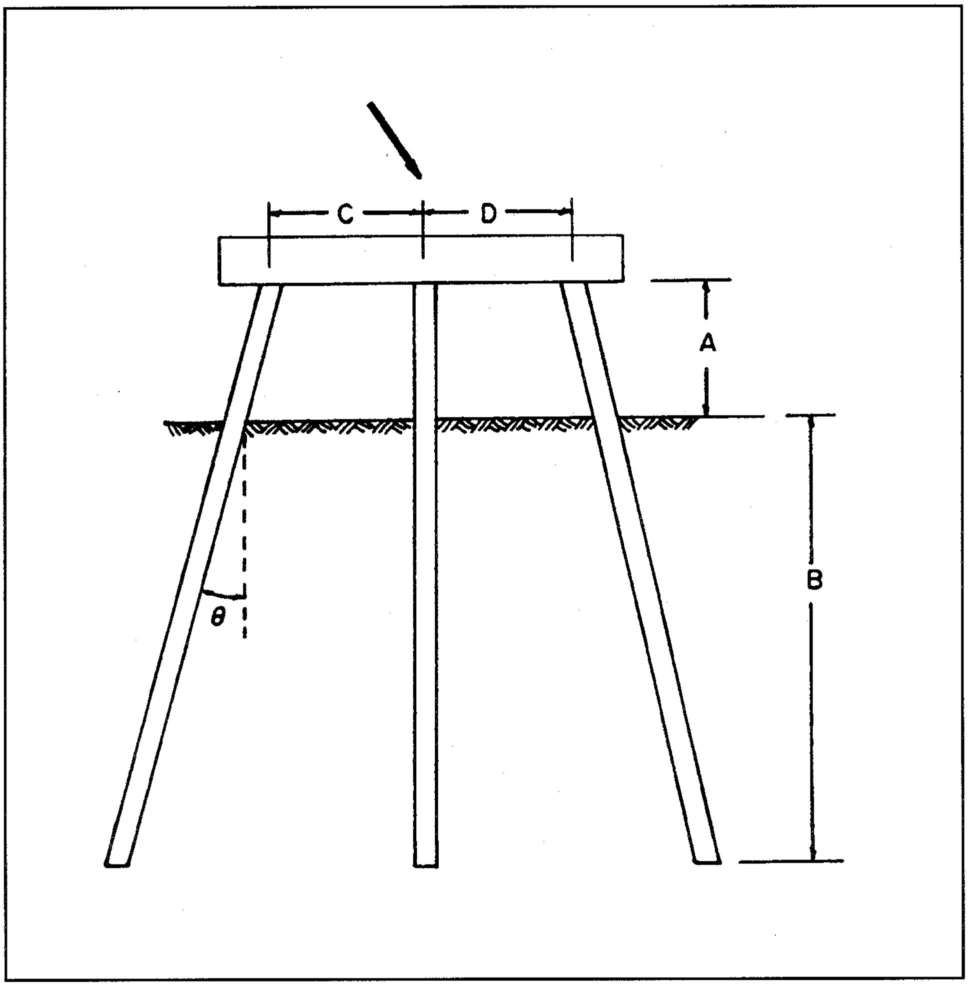


Figure 5-3. Typical pile-supported bent

The assumption is made that curves can be developed, usually nonlinear, that give the relationship between pile-head movement and pile-head forces. A secant to a curve is obtained at the point of deflection and called the modulus of pile-head resistance. The values of the moduli, so obtained, can then be used, as shown below, to compute the components of movement of the structure. If the values of the moduli that were selected were incorrect, iterations are made until convergence is obtained. Using sign conventions established for the single pile under lateral loading, the lateral force P_t at the pile head may be defined as follows:

$$P_t = J_y y_t \tag{5-10}$$

If there is some rotational restraint at the pile-head, the moment is

$$M_t = -J_m y_t \tag{5-11}$$

The moduli J_y and J_m are not single-valued functions of pile-head translation but are functions also of the rotation α_s of the structure. For batter piles, a procedure is given in Appendix D for adjusting values of soil resistance to account for the effect of the batter. If it is assumed that a compressive load causes a positive deflection along the pile axis, the axial force P_x may be defined as follows:

$$P_x = J_x x_t \tag{5-12}$$

It is usually assumed that P_x is a single-valued function of x_t. A curve showing axial load versus deflection may be computed by one of the procedures recommended by several authors (Reese 1964; Coyle and Reese 1966; Coyle and

Sulaiman 1967; Kraft, Ray, and Kagawa 1981) or the results from a field load test may be used. A typical curve is shown in Figure 5-5a.

(3) Computer programs. Computer programs or nondimesional methods may be used to obtain curves showing lateral load as a function of lateral deflection and pile-head moment as a function of lateral deflection. The way the pile is attached to the superstructure must be taken into account in making the computations. Typical curves are shown in Figures 5-5b and 5-5c. The forces at the pile head defined in equations 5-10 through 5-12 may now be resolved into vertical and horizontal components of force on the structure, as follows:

$$F_v = -(P_x \cos\theta - P_t \sin\theta) \tag{5-13}$$

$$F_h = -(P_x \sin\theta + P_t \cos\theta) \tag{5-14}$$

The moment on the structure is

$$M_s = J_m y_t \tag{5-15}$$

The equilibrium equations can now be written, as follows:

$$P_v + \Sigma F_{v_i} = 0 \tag{5-16}$$

$$P_h + \Sigma F_{h_i} = 0 \tag{5-17}$$

$$M + \Sigma M_{s_i} + \Sigma a_i F_{v_i} + \Sigma b_i F_{h_i} = 0 \tag{5-18}$$

The subscript i refers to values from any "i-th" pile. Using equations 5-8 through 5-15, equations 5-16 through 5-18 may be written in terms of the structural movements. Equations 5-19 through 5-21 are in the final form.

$$\begin{aligned} P_v = {} & \Delta v [\Sigma A_i] + \Delta h [\Sigma B_i] \\ & + \alpha_s [\Sigma a_i A_i + \Sigma b_i B_i] \end{aligned} \tag{5-19}$$

$$\begin{aligned} P_h = {} & \Delta v [\Sigma B_i] + \Delta h [\Sigma C_i] \\ & + \alpha_s [\Sigma a_i B_i + \Sigma b_i C_i] \end{aligned} \tag{5-20}$$

$$\begin{aligned} M = {} & \Delta v [\Sigma D_i + \Sigma a_i A_i + \Sigma b_i B_i] \\ & + \Delta h [\Sigma E_i + \Sigma a_i B_i + \Sigma b_i C_i] \\ & + \alpha_s [\Sigma a_i D_i + \Sigma a_i^2 A_i + \Sigma b_i E_i \\ & + \Sigma b_i^2 C_i + \Sigma 2 a_i b_i B_i] \end{aligned} \tag{5-21}$$

where

$$A_i = J_{x_i} \cos^2\theta_i + J_{y_i} \sin^2\theta_i$$

$$B_i = (J_{x_i} - J_{y_i}) \sin\theta_i \cos\theta_i$$

$$C_i = J_{x_i} \sin^2\theta_i + J_{y_i} \cos^2\theta_i$$

$$D_i = J_{m_i} \sin\theta_i$$

$$E_i = -J_{m_i} \cos\theta_i$$

These equations are not as complex as they appear. For example, the origin of the coordinate system can usually be selected so that all of the b-values are zero. For vertical piles, the sine terms are zero and the cosine terms are unity. For small deflections, the J-values can all be taken as constants. Therefore, under a number of circumstances it is possible to solve these equations by hand. However, if the deflections of the group is such that the nonlinear portion of the curves in Figure 5-5 is reached, the use of a computer solution is advantageous. Such a program is available through the Geotechnical Engineering Center, The University of Texas at Austin (Awoshika and Reese 1971; Lam 1981).

(4) Detailed step-by-step solution procedure.

(a) Study the foundation to be analyzed and select a two-dimensional bent where the behavior is representative of the entire system.

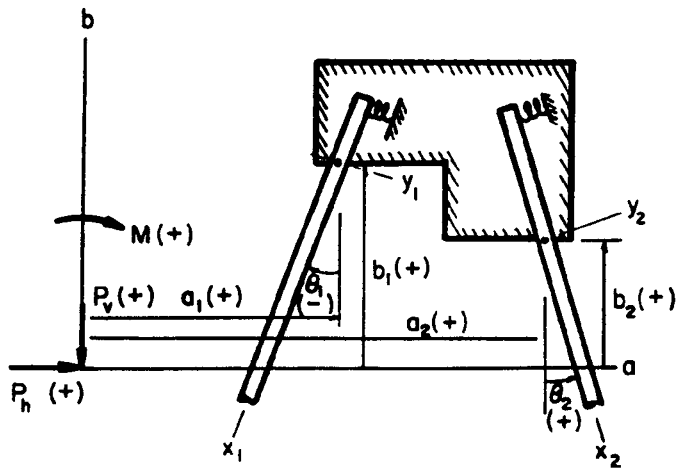
(b) Prepare a sketch such that the lateral loading comes from the left. Show all pertinent dimensions.

(c) Select a coordinate center and find the horizontal component, the vertical component, and the moment through and about that point.

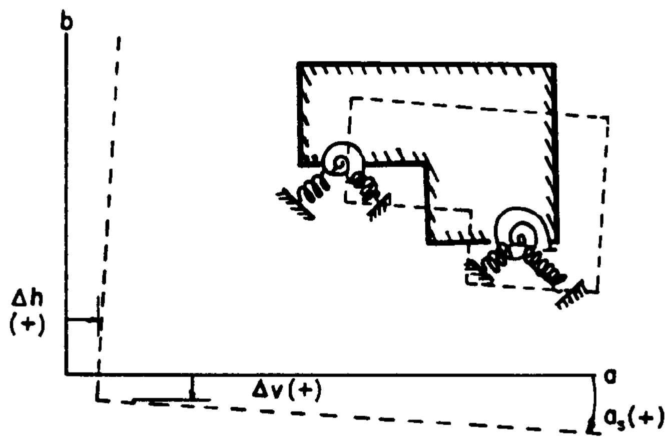
(d) Compute by some procedure a curve showing axial load versus axial deflection for each pile in the group; or, preferably, use the results from a field load test.

(e) Use appropriate procedures and compute curves showing lateral load as a function of lateral deflection and moment as a function of lateral deflection, taking into account the effect of structural rotation on the boundary conditions at each pile head.

(f) Estimate trial values of J_x, J_y, and J_m for each pile in the structure.

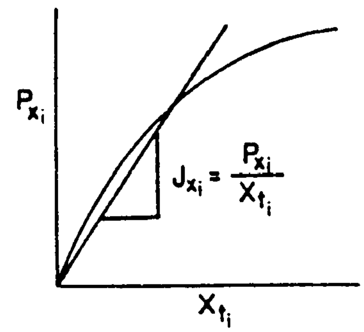


a. With piles shown

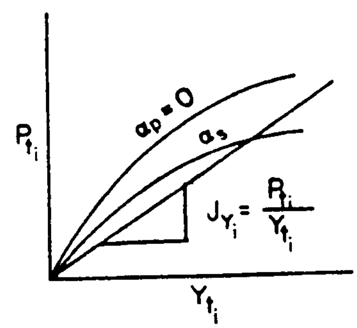


b. With piles represented as springs

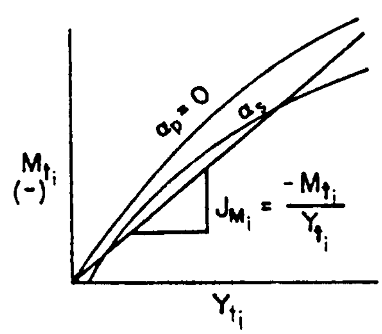
Figure 5-4. Simplified structure showing coordinate systems and sign conventions



a. Axial pile resistance versus axial displacement.



b. Lateral pile resistance versus lateral pile displacement.



c. Moment at pile head versus lateral pile displacement for various rotations (α_p) of the pile head.

Figure 5-5. Set of pile resistance functions for a given pile

(g) Solve equations 5-19 through 5-21 for values of Δv, Δh, and α_s .

(h) Compute pile-head movements and obtain new values of J_x , J_y , and J_m for each pile.

(i) Solve equations 5-19 through 5-21 again for new values of Δv, Δh, and α_s .

(j) Continue iteration until the computed values of the structural movements agree, within a given tolerance, with the values from the previous computation.

(k) Compute the stresses along the length of each pile using the loads and moments at each pile head.

(5) Example problem. Figure 5-6 shows a pile-supported retaining wall with the piles spaced 8 feet apart. The piles are 14 inches in outside diameter with four No. 7 reinforcing steel bars spaced equally. The centers of the bars are on an 8-inch circle. The yield strength of the reinforcing steel is 60 kips per square inch and the compressive strength of the concrete is 2.67 kips per square inch. The length of the piles is 40 feet. The backfill is a free-draining, granular soil

with no fine particles. The surface of the backfill is treated to facilitate a runoff, and weep holes are provided so that water will not collect behind the wall. The forces P_1, P_2, P_s, and P_w (shown in Figure 5-6) were computed as follows: 21.4, 4.6, 18.4, and 22.5 kips, respectively. The resolution of the loads at the origin of the global coordinate system resulted in the following service loads: P_v = 46 kips, P_h = 21 kips, and M = 40 foot-kips (some rounding was done). The moment of inertia of the gross section of the pile was used in the analysis. The flexural rigidity EI of the piles was computed to be 5.56×10^9 pounds per square inch. Computer Program PMEIX was run and an interaction diagram for the pile was obtained. That diagram is shown in Figure 5-7. A field load test was performed at the site and the ultimate axial capacity of a pile was found to be 176 kips. An analysis was made to develop a curve showing axial load versus settlement. The curve is shown in Figure 5-8. The subsurface soils at the site

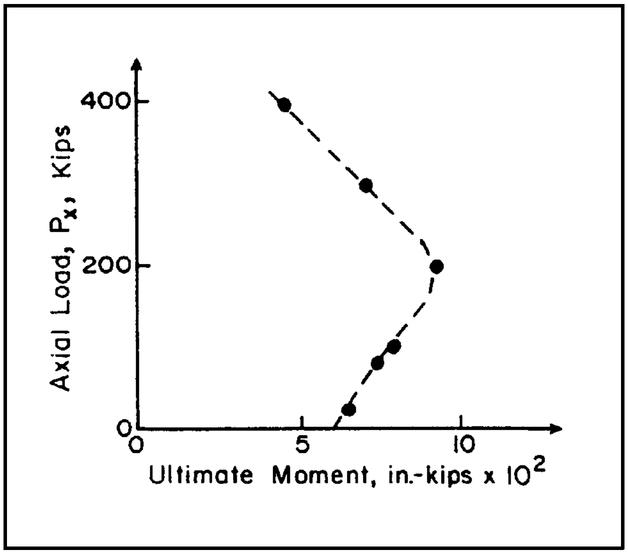


Figure 5-7. Interaction diagram of reinforced concrete pile

consist of silty clay. The water content averaged 20 percent in the top 10 feet and averaged 44 percent below 10 feet. The water table was reported to be at a depth of 10 feet from the soil surface. There was a considerable range in the undrained shear strength of the clay and an average value of 3 kips per square foot was used in the analysis. A value of the submerged unit weight of 46 pounds per cubic foot as employed and the value of ε_{50} was estimated to be 0.005. In making the computations, the assumption was made that all of the load was carried by piles with none of the load taken by passive earth pressure or by the base of the footing. It was further assumed that the pile heads were free to rotate. As noted earlier, the factor of safety must be in the loading. Therefore, the loadings shown in Table 5-3 were used in the preliminary computations. Table 5-4 shows the movements of the origin of the global coordinate system when equation 5-19 through 5-21 were solved simultaneously. The loadings were such that the pile response was almost linear so that only a small number of iterations were

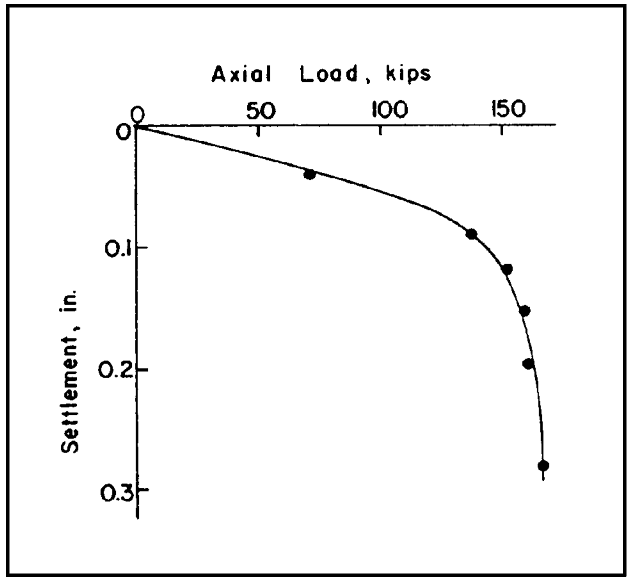


Figure 5-8. Axial load versus settlement for reinforced concrete pile

required to achieve converenge. The computed pile-head movements, loads, and moments are shown in Table 5-5.

(6) Verify results. The computed loading on the piles is shown in Figure 5-9 for Case 4. The following check is made to see that the equilibrium equations are satisfied.

$$\Sigma F_v = 24.2 + 97.2 \cos 14 - 14.3 \sin 14$$
$$= 24.2 + 94.3 - 3.5 = 115.0 \text{ kips OK}$$
$$\Sigma F_h = 15.2 + 14.3 \cos 14 + 97.2 \sin 14$$
$$= 15.2 + 13.9 + 23.6 = 52.7 \text{ kips OK}$$
$$\Sigma M = -(24.2)(1.5) + (97.2 \cos 14)(1.5) - (14.3 \sin 14)(1.5)$$
$$= -36.3 + 141.4 - 5.2$$
$$= 99.9 \text{ ft-kips OK}$$

Table 5-3
Values of Loading Employed in Analyses

Case	Loads, kips		moment, ft-kips	Comment
	P_v	P_h		
1	46	21	40	service load
2	69	31.5	60	1.5 times service load
3	92	42	80	2 times service load
4	115	52.5	100	2.5 times service load

Note: $P_v / P_h = 2.19$

Table 5-4
Computed Movements of Origin of Global Coordinate System

Case	Vertical movement Δv	Horizontal movement Δh	Rotation α
	in.	in.	rad
1	0.004	0.08	9×10^{-5}
2	0.005	0.12	1.4×10^{-4}
3	0.008	0.16	1.6×10^{-4}
4	0.012	0.203	8.4×10^{-5}

Thus, the retaining wall is in equilibrium. A further check can be made to see that the conditions of compatibility are Figure 5-8, an axial load of 97.2 kips results in an axial deflection of about 0.054 inch, a value in reasonable satisfied. One check can be made at once. Referring to agreement with the value in Table 5-5. Further checks on compatibility can be made by using the pile-head loadings and Computer Program COM622 to see if the computed deflections under lateral load are consistent with the values tabulated in Table 5-5. No firm conclusions can be made concerning the adequacy of the particular design without further study. If the assumptions made in performing the analyses are appropriate, the results of the analyses show the foundation to be capable of supporting the load. As a matter of fact, the piles could probably support a wall of greater height.

c. Closely spaced piles. The theory of elasticity has been employed to take into account the effect of a single pile on others in the group. Solutions have been developed (Poulos 1971; Banerjee and Davies 1979) that assume a linear response of the pile-soil system. While such methods are instructive, there is ample evidence to show that soils cannot generally be characterized as linear, homogeneous, elastic materials. Bogard and Matlock (1983) present a method in which the *p*-*y* curve for a single pile is modified to take into account the group effect. Excellent agreement was obtained between their computed results and results from field experiments (Matlock et al. 1980). Two approaches to the analysis of a group of closely spaced piles nder lateral load are given in the following paragraphs. One method is closely akin to the use of efficiency formulas, and the other method is based on the assumption that the soil within the pile group moves laterally the same amount as do the piles.

(1) Efficiency formulas. Pile groups under axial load are sometimes designed by use of efficiency formulas. Such a formula is shown as equation 5-22.

$$(Q_{ult})_G = E \times n \times (Q_{ult})_p \qquad (5\text{-}22)$$

where

$(Q_{ult})_G$ = ultimate axial capacity of the group

E = efficiency factor (1 or < 1)

n = number of piles in the group

$(Q_{ult})_p$ = ultimate axial capacity of an individual pile

Various proposals have been made about obtaining the value of E; for example, McClelland (1972) suggested that the value of E should be 1.0 for pile groups in cohesive soil with center-to-center spacing of eight diameters or more and that E should decrease linearly to 0.7 at a spacing of three diameters. McClelland based his recommendations on results from experiments in the field and in the laboratory. It is of interest to note that no differentiation is made between piles that are spaced front to back, side by side, or spaced as some other angle between each other. Unfortunately, experimental data are limited on the behavior of pile groups under lateral load. Furthermore, the mechanics of the behavior of a group of laterally loaded piles are more complex than for a group of axially loaded piles. Thus, few recommendations have been made for efficiency formulas for laterally loaded groups. Two different recommendations have been made regarding the modification of the coefficient of subgrade reaction. The Canadian Foundation Engineering Manual (Canadian Geotechnical Society 1985) recommends that the coefficient of subgrade reaction for pile groups be equal to that of a single pile if the spacing of the piles in the group is eight diameters. For spacings smaller than eight diameters, the following ratios of the single-pile subgrade reaction were recommended: six diameters, 0.70; four diameters, 0.40; and three diameters, 0.25. The Japanese Road Association (1976) is less conservative. It is suggested that a slight reduction in the coefficient of horizontal subgrade reaction has no serious effect with regard to bending stress and that the use of a factor of safety should be sufficient in design except in the case where the piles get quite close together. When piles are closer together than two and one-half diameters, the following equation is suggested for computing a factor μ to multiply the coefficient of subgrade reaction for the single pile.

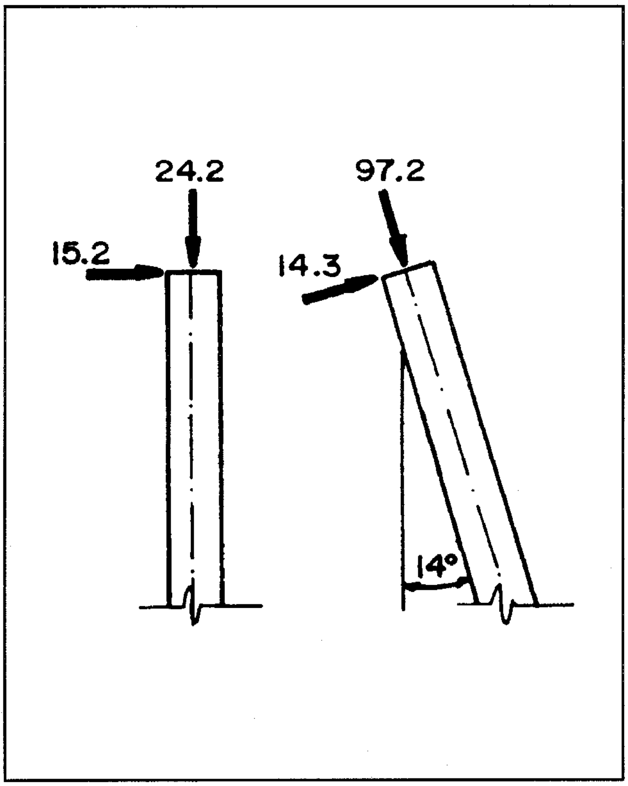


Figure 5-9. Pile loading - Case 4

$$\mu = 1 - 0.2(2.5 - L/D) \quad L < 2.5D \qquad (5\text{-}23)$$

where

L = center-to-center distance between piles

D = pile diameter

Table 5-5
Computed Movements and Loads at Pile Heads

Case	Pile 1					Pile 2				
	x_t	y_t	P_x	P_t	M_{max}	x_t	y_t	P_x	P_t	M_{max}
	in.	in.	kips	kips	in. - kips	in.	in.	kips	kips	in. -kips
1	0.005	0.08	9.7	6.0	148	0.02	0.077	38.9	5.8	143
2	0.008	0.12	14.5	9.0	222	0.03	0.116	58.3	8.6	215
3	0.011	0.162	19.3	12.1	298	0.04	0.156	77.7	11.5	288
4	0.013	0.203	24.2	15.2	373	0.06	0.194	97.2	14.3	360

(2) Single-pile method. The single-pile method of analysis is based on the assumption that the soil contained between the piles moves with the group. Thus, the pile group that contained soil can be treated as a single pile of large diameter.

(3) A step-by-step procedure for single-pile method.

(a) The group to be analyzed is selected and a plan view of the piles at the groundline is prepared.

(b) The minimum length is found for a line that encloses the group. If a nine-pile (three by three) group consists of piles that are 1 foot square and three widths on center, the length of the line will be 28 feet.

(c) The length found in step b is considered to be the circumference of a pile of large diameter; thus, the length is divided by π to obtain the diameter of the imaginary pile having the same circumference of the group.

(d) The next step is to determine the stiffness of the group. For a lateral load passing through the tops of the piles, the stiffness of the group is taken as the sum of the stiffness of the individual piles. Thus, it is assumed that the deflection at the pile top is the same for each pile in the group and, further, that the deflected shape of each pile is identical. Some judgment must be used if the piles in the group have different lengths.

(e) Then, an analysis is made for the imaginary pile, taking into account the nature of the loading and the boundary conditions at the pile head. The shear and moment for the imaginary large-size pile is shared by the individual piles according to the ratio of the lateral stiffness of the individual pile to that of the group.

The shear, moment, pile-head deflection, and pile-head rotation yield a unique solution for each pile in the group. As a final step, it is necessary to compare the single-pile solution to that of the group. It could possibly occur that the piles in the group could have an efficiency greater than one, in which case the single-pile solutions would control.

(4) Example problem. A sketch of an example problem is shown in Figure 5-10. It is assumed that steel piles are embedded in a reinforced concrete mat in such a way that the pile heads do not rotate. The piles are 14HP89 by 40 feet long and placed so that bending is about the strong axis. The moment of inertia is 904 inches^4 and the modulus of elasticity of 30 × 10^6 pounds per square inch. The width of the section is 14.7 inches and the depth is 13.83 inches. The soil is assumed to be a sand with an angle of internal friction of 34 degrees, and the unit weight is 114 pounds per cubic foot. The computer program was run with a pile diameter of 109.4 inches and a moment of inertia of 8,136 inches^4 (nine times 904). The results were as follows:

$$y_t = 0.885 \text{ inch}$$

$$M_t = M_{max} = 3.60 \times 10^7 \text{ in.-lb for group}$$

$$= 3.78 \times 10^6 \text{ in.-lb for single pile}$$

$$\text{Bending stress} = 25.3 \text{ kips/sq in.}$$

The deflection and stress are for a single pile. If a single pile is analyzed with a load of 50 kips, the groudline deflection was 0.355 inch and the bending stress was 23.1 kips per square inch. Therefore, the solution with the imaginary large-diameter single pile was more critical.

5. Computer Assisted Analysis

A computer assisted analysis is a reasonable alternative for obtaining reliable estimates of the performance of pile groups. Several computer programs can assist the analysis and design of groups.

a. CPGA. Program CPGA provides a three-dimensional stiffness analysis of a group of vertical and/or battered piles assuming linear elastic pile-soil interaction, a rigid pile cap, and a rigid base (WES Technical Report ITL-89-3). Maxtrix methods are used to incorporate position and batter of piles as well as piles of different sizes and materials. Computer program CPGG displays the geometry and results of program CPGA.

b. STRUDL. A finite element computer program such as STRUDL or SAP should be used to analyze the performance of a group of piles with a flexible base.

c. CPGC. Computer program CPGC develops the interaction diagrams and data required to investigate the structural capacity of prestressed concrete piles (WES Instruction Report ITL-90-2).

d. CPGD. Computer program (Smith and Mlakar 1987) extends the rigid cap analysis of program CPGA to provide a simplified and realistic approach for seismic analysis of pile foundations. Program CPGD (in development stage at WES) includes viscous damping and response-spectrum loading to determine pile forces and moments.

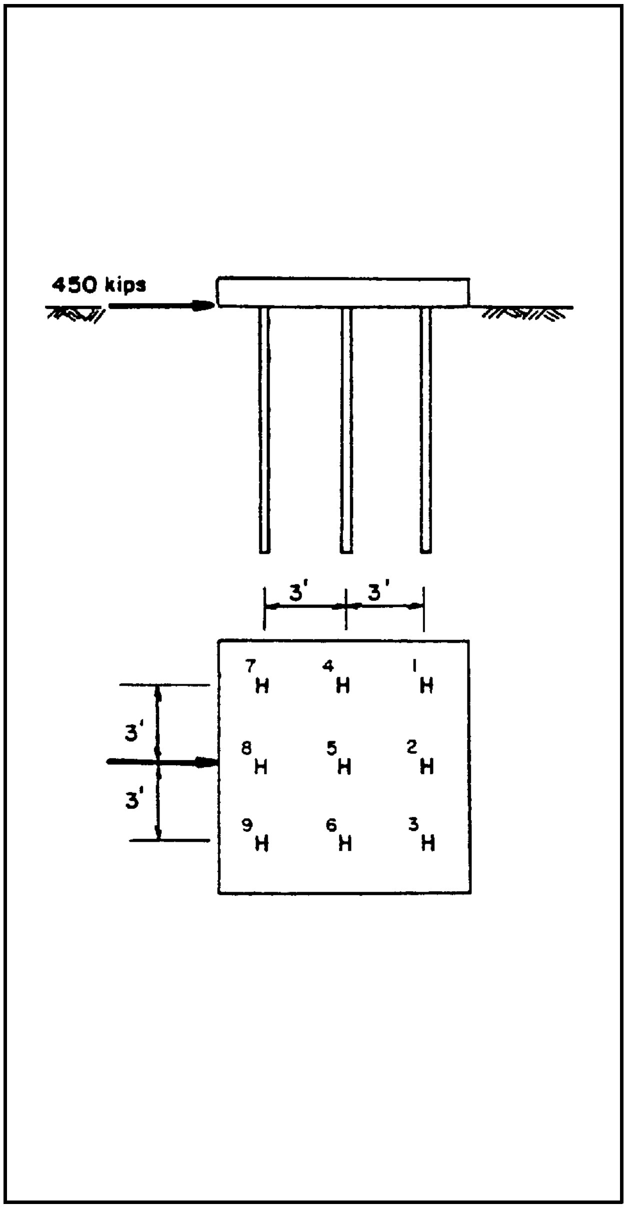


Figure 5-10. Plan and evaluation of foundation analyzed in example problem